EECS C145B / BioE C165 Spring 2003: Problem Set I Due February 14 2003

Please read the sections describing the rules for working in groups and the grading policy in the course introduction handout.

Problem 1

Are the following systems linear, spatially invariant, both or neither? Assume g is the system output and f is the system input.
Hint: Ask yourself the following questions:

1. If I scale the input by k, will the output be scaled by k?
2. Will a shift in the input cause only a shift in the output?
3. Imagine that the input is a sinusoid. Does the system produce an output that has other frequency components in it?

1. $g(x) = 2f(x)$
2. $g(x, y) = f(x - 2, y - 6) + 3f(x + 3, y - 3)$
3. $g(x, y) = f(x - 2, y - 6) + 3f(x + 3, y - 3) + 5$
4. $g(x, y, z) = 3f(x, y, z - 5.5)$
5. $g(x, y, z) = 10$
6. $g(x) = f^2(x)$

7. $g(x,y) = \frac{df(x,y)}{dx}$

8. $g(x) = f(x-5) + f(3x)$

9. $g[m,n] = f[m-2, n-6] + f[m+3, n-3]$

10. $g(x,y) = x + y$

11. $g(x,y) = x^2 + y^2$

(2 points each = 20 + 2 bonus)

Problem 2

Convolve the following images f with the point spread functions h, by hand, showing all working:

1. $f[m] = \delta[m-2] + \delta[m-1]$, $h[m] = \delta[m] + \delta[m-1]$
2. $$f[m,n] = \begin{bmatrix} 1 & 5 & -1 \\ 8 & 0 & 3 \\ 9 & 7 & 8 \end{bmatrix}$$
$$h[m,n] = \delta[m,n] - \delta[m,n-1] - 2\,\delta[m-1,n] + 0\,\delta[m-1,n-1]$$
3. $$f[m,n,0] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$f[m,n,1] = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$
$$h[m,n,0] = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$
$$h[m,n,1] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
Note: This is a 3D convolution.

(10, 15, 10+5 bonus)

Problem 3

Find the following DFTs by hand (you can check them for correctness using Matlab).

1. $f[m] = [1\ 0\ 0\ 0]$
2. $f[m] = [1\ 0\ -1\ 0]$
3. $$f[m,n] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
4. Calculate the magnitude and phase matrices for your answer to (3) above. Show all calculations.

(10,10,20,10)

Problem 4

1. Show that the inverse Fourier transform of:
$$F(u,v) = \mathrm{rect}(u/2, v/2)$$
is
$$f(x,y) = 4\,\mathrm{sinc}(2x, 2y)$$
Hint: Take advantage of the separability property of the FT. Also recall:
$$\sin(\theta) = \frac{1}{2j}\left(e^{\theta} - e^{-\theta}\right)$$
2. In 1D the sinc function has the special property that it contains all frequency components equally up to a cutoff. Why is this not true in 2D?

(15, 2 bonus)

Problem 5

You have been asked to simulate the acoustics of a cavern in a cave. You are standing at point A. Two large flat walls stand at -45 and 45 with respect to the direction you are facing. They are both 20 meters away. Directly behind you is another large wall. Your laser distance finder tells you it is 50 meters away. In between these walls there are tall passageways opening into larger caverns. Your rangefinder reads ∞ when aimed into these spaces. The ceiling is 100 meters above the ground.
Model the acoustics of the cavern using a linear shift invariant system. Draw the structure of the cave. Plot the impulse response of the system. Label the time and amplitude axes. Assume the sound is emitted and recorded on the ground at point A, and radiates equally in all directions. Assume all surfaces reflect sound with equal efficiency. The speed of sound in the cave is 340 m/s.
(25 points)

Problem 6

A 640×480 pixel TV image suffers from power line interference when a vacuum cleaner is brought near the TV set. The interference takes the form of vertical lines, equally spaced. You observe 11 full periods of these bars across the screen. You capture the image and take its DFT and find the magnitude spectrum. You do not shift the spectrum to place zero frequency at the center. At what sample numbers of the DFT are the peaks of the spectral components due to the interference observed? Remember, if u_s is the sampling frequency and N is the number of samples along the u-axis, then the axis of the DFT is $0, u_s/N, 2u_s/N, \ldots, (N-1)u_s/N$. Recall also the conjugate symmetry of the DFT.
(20 points + 5 bonus)

Problem 7

After reading Section 4.6.6 of Gonzalez and Woods (pp. 208-213), answer the following questions.

1. What is the FFT used for?

2. What are the differences between the DFT and the FFT?

3. How are computational savings achieved in the FFT?

4. What length DFTs are most suitable for implementation using the FFT?

5. Gonzalez and Woods is a textbook on image processing. Why do they not derive a 2D FFT?

6. One of your colleagues implements the DFT in C directly from its definition. He asks you to predict how the speed of his code will compare to Matlab's FFT function when computing a 16 point DFT. Approximately how many times faster will the Matlab code be?

(20 points)

Problem 8

Show the structure of a matrix, which, when multiplied with a vector, gives the 1D DFT of that vector.
(10 points)